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Boundary states for D-branes with traveling waves

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Abstract: We construct boundary states for D-branes which carry traveling waves in the covariant formalism. We compute their vacuum amplitudes to investigate their interactions. In non-compact space, the vacuum amplitudes become trivial as is common in plane wave geometries. However, we found that if they are compactified in the traveling direction, then the amplitudes are affected by non-trivial time dependent effects. The interaction between D-branes with waves traveling in the opposite directions ('pulse-antipulse scattering') are also computed. Furthermore, we apply these ideas to open string tachyon condensation with traveling waves.

Keywords: Superstrings and Heterotic Strings, D-branes, Conformal Field Models in String Theory.

Contents

1. Introduction

String theory on plane wave geometries [1] has many interesting features. In particular, the maximally supersymmetric plane wave solution to type IIB supergravity is found in [2], and superstrings on this background can be exactly solved in the light-cone Green-Schwarz formalism even in the presence of RR-field [3]. This background attracts much attention also because we can discuss its Yang-Mills theory dual [4].

A plane wave in D dimensional spacetime is generally defined by the metric (in the Brinkman coordinate)

$$ds^2 = -2dx^+dx^- - \sum_{i,j=1}^{D-2} h_{ij}(x^+)x^ix^j(dx^+)^2 + \sum_{i=1}^{D-2}(dx^i)^2\,, \tag{1.1}$$

and it is time dependent via the term $h_{ij}(x^+)$. Physically, this term represents the traveling gravitational waves. Furthermore, the background preserves at least a half of supersymmetries. Thus, it may lead to a solvable time dependent model with supersymmetry [5]. (Refer to [6, 7] for null orbifolds, which have similar properties.)

However, we have also a disadvantage that we do not know well how to quantize covariantly string theory in general plane waves.[1] For example, it is not completely unambiguous how to compute even the cosmological constant (i.e., vacuum amplitude).

Motivated by these observations, we would like to discuss the open string analogue of strings on plane waves. In particular, we consider the D-branes with traveling waves in flat space; either waves of gauge fields $A^i(x^+, x^i)$ or transverse scalar fields $\phi^I(x^+, x^i)$, which are related to each other by T-duality. Indeed, we can show that open string metric on a D-brane with such gauge fields leads to a metric of a pp-wave (or plane wave if we choose the specific profiles of $A^i(x^+, x^i)$) and that it is a 1/4 BPS state. We can choose any functions of x^+ as gauge fields or scalar fields while preserving boundary conformal symmetry as noticed in [11, 12, 13]; for instance, we can consider a D-brane with a pulse-like worldvolume. Recently, this configuration was examined in the nice paper [14] (see also [15] for more generalized models) in the light-cone gauge of open string, where the world-sheet theory is manifestly time dependent. There are also some earlier discussions on the related or analogous backgrounds, see [16, 17] for strings with traveling waves, and [12, 18, 19] for null intersection of D-branes.

It is useful to apply the covariant quantization in order to extract information intrinsic to the time dependent physics, and hence we construct the boundary states representing the D-branes with traveling waves in the covariant formalism and examine their properties. These are new type of boundary states in flat spacetime with infinite parameters. Furthermore, we compute vacuum amplitudes in non-compact and compactified flat spacetime, where the closed string theory is very simple. In the non-compact case we find a rather trivial result and indeed it is the same as that of usual D-brane. This means that the interaction between the D-branes is the same as the usual D-branes, which is consistent with the fact that there is no vacuum polarization in plane wave background [20, 21]. On the other hand, if we compactify the traveling direction, then we obtain a very non-trivial amplitude, which reflects the time dependence of traveling waves. We also compute the interaction between two waves (or pulses) traveling in the opposite directions. It is possible to calculate it only in the covariant formalism. We argue that the collision might lead to open string pair creation like [22] as well as open string tachyonic modes. Finally, we apply these methods to the open string tachyon condensation [23]. A configuration with traveling open string tachyon is considered, and the corresponding boundary state is constructed.

The organization of this paper is as follows. In section 2 we construct the boundary state for the D-brane with traveling waves and compute the vacuum amplitude. By using this boundary state, we also compute the energy momentum tensor. In section 3 we consider the D-branes with traveling waves wrapped on a circle and compute the vacuum amplitude.

[1] In particular, superstrings on time *in*dependent plane waves (i.e., constant h_{ij}) with NSNS-flux can be described by Nappi-Witten model [8], therefore we can quantize the superstrings covariantly. Recent developments are given in [9]. See also [10].

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In section 4 we discuss more general configuration with traveling waves depending also on the other world-volume coordinates. In section 5 we consider the interaction between two waves traveling in the opposite directions. In section 6 we discuss the application to open string tachyon condensation. In section 7 we give a brief summary of our results and draw conclusions.

2. Boundary states for D-branes with traveling waves

There are gauge fields A^i and transverse scalar fields ϕ^I on D-branes as massless bosonic fields. The time dependent expectation values of these fields give interesting time dependent D-brane backgrounds. Since there is a duality among open string and closed string in string theory, this will also lead to an intriguing influence on closed strings. We would like to investigate this issue in simple examples of D-branes with traveling waves. In this background, gauge fields or transverse scalar fields depend only on x^+ and describes waves on D-branes which are traveling at the speed of light. Interestingly the profiles of waves can be chosen arbitrarily as we explain in the boundary state formalism. Intuitively we can understand this by the fact that the operators included in $A^i(X^+)$ and $\phi^I(X^+)$ (or α_n^+) have no non-trivial commutation relations with each other [11, 12, 14]. Below we construct boundary states for D-branes with gauge field waves $A^i(x^+)$ and discuss their physical properties. The results for D-branes with transverse scalar waves can be obtained by T-duality. More general forms of gauge fields will be considered in section 4.

2.1 Preparations and conventions

In this paper we define the mode expansion of closed string in non-compact spacetime as

$$X^\mu(\tau,\sigma) = x^\mu + 2\alpha' p^\mu \tau + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\left(\alpha_n^\mu e^{-2in(\tau+\sigma)} + \tilde{\alpha}_n^\mu e^{-2in(\tau-\sigma)}\right), \tag{2.1}$$

where the closed string $X^\mu(\tau,\sigma)$ has the periodicity under $\sigma \to \sigma+\pi$. The commutation relations are

$$[\alpha_m^\mu, \alpha_n^\nu] = [\tilde{\alpha}_m^\mu, \tilde{\alpha}_n^\nu] = m\eta^{\mu\nu}\delta_{m,-n}\,, \qquad [x^\mu, p^\nu] = i\eta^{\mu\nu}\,, \tag{2.2}$$

and the other commutators vanish. In compactified cases (on a rectangular torus with radii R^μ) we should add winding term $2R^\mu w_\mu \sigma$ to the mode expansion (2.1).

We define the coherent state $|x\rangle$ (including only massive modes [24]), which is useful to construct the boundary state with non-zero gauge flux, as follows (here we suppress the index μ in α_n^μ)

$$|x\rangle = \prod_{m\geq 1}\exp\left[\frac{1}{m}\alpha_{-m}\tilde{\alpha}_{-m} + \frac{x_m}{m}\alpha_{-m} - \frac{x_{-m}}{m}\tilde{\alpha}_{-m} - \frac{x_m x_{-m}}{2m}\right]|0\rangle\,, \tag{2.3}$$

where we impose $x_m = x^*_{-m}$. This state satisfies

$$(\alpha_n - \tilde{\alpha}_{-n} - x_n)|x\rangle = 0\,. \tag{2.4}$$

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Then the Dirichlet boundary state $|D,\vec{x}\rangle$ located at $\vec{x}$ is simply given by $|x=0\rangle\otimes|\vec{x}\rangle_{(0)}$. The state $|\vec{x}\rangle_{(0)}$ means the zero-mode part of the boundary state and it is normalized such that $\langle\vec{x}|\vec{x'}\rangle_{(0)}=\delta(\vec{x}-\vec{x'})$. The Neumann boundary state, i.e., D25-brane, is given by the integral

$$\begin{aligned}|N\rangle &= \frac{T_{25}}{2}\int d\vec{x}\prod_{m\geq 1}\frac{dx_m dx_{-m}}{2\pi m}\ |x\rangle\otimes|\vec{x}\rangle_{(0)}\\ &= \frac{T_{25}}{2}\exp\left(-\sum_{m=1}^{\infty}\frac{\alpha_{-m}\tilde{\alpha}_{-m}}{m}\right)\otimes\int d\vec{x}\ |\vec{x}\rangle_{(0)}\,,\end{aligned} \tag{2.5}$$

where the normalization constant T_p (for the Dp-brane) is given by $T_p=2^{7-p}\pi^{\frac{23}{2}-p}\alpha'^{\frac{11-p}{2}}$. The normalization T_p is fixed by making use of the open-closed duality (Cardy's condition [25]). In other words, we have[2]

$$\langle N|\Delta|N\rangle=\int\frac{dt}{2t}\,\mathrm{Tr}[e^{-2\pi tH_o}]\,, \tag{2.6}$$

where H_o is the open string hamiltonian. In the compactified case we only have to replace $|\vec{x}\rangle_{(0)}$ with $\sum_{w=-\infty}^{\infty}|\vec{x},w\rangle_{(0)}$, where w is the winding number of the compactified direction.

We should mention that even though in this paper we always use boundary states in the (Lorentz) covariant formalism (for a review, see [26]), we suppress the ghost part because it has the usual form

$$|\mathrm{ghost}\rangle=\exp\biggl(-\sum_{n=1}^{\infty}(\tilde{b}_{-n}c_{-n}+b_{-n}\tilde{c}_{-n})\biggr)(c_0+\bar{c}_0)c_1\tilde{c}_1|0\rangle_{\mathrm{SL}(2,\mathbb{R})}\,. \tag{2.7}$$

The generalizations to the similar boundary states in superstring theory are also possible in a rather straightforward way (see, e.g., [24, 27, 28, 26]). Although in this paper we will mainly show the calculations in bosonic string theory, most of the results can be easily extended to the superstring cases (as we will mention later).[3] Thus we omit the details of boundary states in superstring theory for simple expressions.

2.2 Construction of boundary states

Now we would like to construct the boundary states for D-branes with traveling waves. For simplicity we will mainly consider the spacetime filling D-brane (D25-brane) with gauge fields $A_i(X^+)$ $(i=1,2,\ldots,24)$ in our arguments below. The corresponding boundary state, which is denoted as $|P\rangle$, satisfies the following boundary conditions[4]

$$\begin{aligned}\partial_\tau X^+|P\rangle &= 0\,,\\ (\partial_\tau X^- - 2\pi\alpha'\partial_+A_i(X^+)\partial_\sigma X^i)|P\rangle &= 0\,,\\ (\partial_\tau X^i - 2\pi\alpha'\partial_+A_i(X^+)\partial_\sigma X^+)|P\rangle &= 0 \quad \text{at} \quad \tau=0\,.\end{aligned} \tag{2.8}$$

[2]Here the trace includes the Chan-Paton degrees of freedom (factor 2 corresponding to the orientations of open string) and zero mode integration as well as the trace over string oscillators.

[3]In the supersymmetric case the normalization is given by $\frac{T_p}{2}=2^{2-p}\pi^{\frac{7}{2}-p}\alpha'^{\frac{3-p}{2}}$, which is defined by $|N\rangle=\frac{T_p}{2}\frac{1+(-1)^F}{2}|x\rangle\otimes|\psi\rangle$.

[4]Here the gauge field is normalized such that $B+2\pi\alpha' F$ is the gauge invariant combination.

Naively, we can construct the boundary state $|P\rangle$ by multiplying the Neumann boundary state $|N\rangle$ by the Wilson line, i.e.,

$$|P\rangle = \mathrm{P}\exp\Big(-i\int_0^\pi \mathrm{d}\sigma A_i(X^+)\partial_\sigma X^i\Big)|N\rangle, \tag{2.9}$$

where P denotes the path ordering.

The boundary state must satisfy the boundary conformal invariance

$$(L_n - \tilde{L}_{-n})|P\rangle = 0 \quad \text{for } {}^\forall n\,, \tag{2.10}$$

where L_n and $\tilde{L}_n$ are Virasoro generators on the flat background. Formally it is easy to show that $|P\rangle$ satisfies eq. (2.10). However, we should take a great care since the naive expression (2.9) would be divergent. We can avoid the divergence by using the renormalization scheme, however it breaks boundary conformal symmetry in general.

Therefore, we should check whether there are divergences or not in the formal expression (2.9), and we can easily examine it by using the boundary state in the path integral formalism [24]. Expanding the gauge field as[5]

$$A^i(X^+) = \sum_{n=-\infty}^{\infty} e^{-2in\sigma}A^i_n(X^+), \quad A^i_n = A^{i*}_{-n}\,, \tag{2.11}$$

we find the path integral expression and obtain the final result after the integration

$$\begin{aligned}|P\rangle \;=\; &\frac{T_{25}}{2}\int d\vec{x}\prod_{m\geq 1}\left(\frac{1}{2\pi m}\right)^{26} dx^\mu_m dx^\mu_{-m}\exp\Big[-i\pi\sqrt{2\alpha'}\sum_{n\neq 0}x^i_n A^i_{-n}(\hat{x}^+)\Big]\times\\ &\times\exp\Bigg[-\frac{1}{m}\Big(\alpha^+_{-m}\tilde{\alpha}^-_{-m}+\alpha^-_{-m}\tilde{\alpha}^+_{-m}+x^-_m\alpha^+_{-m}-x^-_{-m}\tilde{\alpha}^+_{-m}+x^+_m\alpha^-_{-m}-x^+_m\tilde{\alpha}^-_{-m}-\\ &\qquad-\frac{x^+_m x^-_{-m}+x^-_m x^+_{-m}}{2}\Big)\Bigg]\exp\Big[\frac{1}{m}\alpha^i_{-m}\tilde{\alpha}^i_{-m}+\frac{x^i_m}{m}\alpha^i_{-m}-\frac{x^i_{-m}}{m}\tilde{\alpha}^i_{-m}-\\ &\qquad\qquad-\frac{x^i_m x^i_{-m}}{2m}\Big]|0\rangle\otimes|\vec{x}\rangle_{(0)}\,,\end{aligned} \tag{2.12}$$

where $A^i_{-n}(\hat{x}^+)$ is defined by $A^i_{-n}(X^+)$ with $X^+(\sigma)$ replaced by

$$\hat{x}^+(\sigma) = x^+ + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}x^+_n e^{-2in\sigma}\,. \tag{2.13}$$

Since $A^i(X^+)$ does not depend on x^-_m, we can integrate over x^-_m and obtain the delta functions $\delta(x^+_m)$. Therefore we get the finite result after the integration over x^+_m (or equally only zero-mode integral $\int dx^+_0$). Then we find that the state (2.9) satisfies the boundary conformal invariance (2.10) including renormalization. We can also understand it in the context of the boundary conformal field theory (see appendix A). It is also straightforward to construct the similar boundary state in superstring theory.

[5]We used the periodicity of A^i under $\sigma \to \sigma + \pi$.

2.3 Energy-momentum tensor from boundary state

In this subsection, we compute the energy-momentum tensor and B-field charge from our boundary state and compare them with the results obtained in [14] as a consistency check. As discussed in [29] if we expand a boundary state $|B\rangle$ for Dp-brane as

$$|B\rangle \propto \int d^{26}k \Big[A_{\mu\nu}(k)\alpha^{\mu}_{-1}\tilde{\alpha}^{\nu}_{-1} + B(k)(b_{-1}\tilde{c}_{-1} + \tilde{b}_{-1}c_{-1})\Big]|k\rangle\,, \tag{2.14}$$

then we can read the value of energy-momentum tensor $T_{\mu\nu}$ as

$$T_{\mu\nu} = K(A_{\mu\nu} + \eta_{\mu\nu}B)\,, \tag{2.15}$$

where K is a constant. The B-field charge $Q_{\mu\nu}$ corresponds to the antisymmetric part of $A_{\mu\nu}$.

Let us apply this method to our boundary state (2.12). It is convenient to use the oscillator representation of $|P\rangle$ obtained by performing integral in eq. (2.12); that is

$$\begin{aligned}|P\rangle = \exp\Bigg(\sum_{m=1}^{\infty}\Big(&-4\pi^2\alpha' m A^i_m(X^+)A^i_{-m}(X^+) - 2\sqrt{2\alpha'}\pi i A^i_m(X^+)\alpha^i_{-m} +\\ &+ 2\sqrt{2\alpha'}\pi i A^i_{-m}(X^+)\tilde{\alpha}^i_{-m}\Big)\Bigg)|N\rangle\,.\end{aligned} \tag{2.16}$$

Expanding $A^i(X^+)$ around $x^+ + 2\alpha' p^+\tau$ and dropping p^+ by using $p^+|N\rangle = 0$, we obtain

$$A^i_m(X^+)|N\rangle = \left[i\sqrt{\frac{\alpha'}{2}}\partial_+ A^i(x^+)\frac{(\alpha^+_m - \tilde{\alpha}^+_{-m})}{m} + O(\alpha_n^{+2})\right]|N\rangle\,. \tag{2.17}$$

Thus we find that the non-zero components are given by

$$\begin{aligned}B &= -\frac{T_{25}}{2}\,, \quad A_{+-} = \frac{T_{25}}{2}\,, \quad A_{ij} = -\frac{T_{25}}{2}\delta_{ij}\,, \quad A_{++} = T_{25}4\pi^2\alpha'^2(\partial_+ A^i)^2\,,\\ A_{+i} &= -A_{i+} = T_{25}2\pi\alpha'\partial_+ A^i \qquad (i,j = 1,2,\ldots,24)\,,\end{aligned} \tag{2.18}$$

and hence the energy momentum tensor is

$$T_{++} = T_{25}4\pi^2\alpha'^2(\partial_+A^i)^2\,, \qquad T_{+-} = T_{25}\,, \qquad T_{ij} = -T_{25}\delta_{ij}\,, \qquad Q_{+i} = T_{25}2\pi\alpha'\partial_+A^i\,, \tag{2.19}$$

where we set $K = 1$ such that the value of energy momentum tensor with $A^i = 0$ agrees with that on the flat background. Performing T-duality, we can show that the results reproduce the ones in [14] computed by using DBI action.

2.4 Vacuum amplitude in non-compact space

One of the most interesting physical properties we can read from the boundary states is the interaction between these D-branes. This can be computed as the vacuum amplitude, which can be directly calculated in our boundary state formalism. As we will see below the result turns out to be rather trivial in the non-compact spacetime.

Let us consider the amplitude between two spacetime filling D-branes with traveling waves of gauge fields $A_i^{(1)}(x^+)$ and $A_i^{(2)}(x^+)$. We only have to evaluate the cylinder amplitude

$$Z = \langle P^{(2)}|\Delta|P^{(1)}\rangle \,, \tag{2.20}$$

where $|P^{(1)}\rangle$ and $|P^{(2)}\rangle$ represent the boundary states of the form (2.12) for two different D-branes. We have also defined the propagator of closed string as

$$\Delta = \frac{\alpha'}{2}\int_0^\infty ds \; e^{-sH_{cl}} \,, \tag{2.21}$$

where H_{cl} denotes the closed string hamiltonian. Then it is easy to see that the massive oscillators α^+_{-n} $(n \geq 1)$ included in $A^i(X^+)$ do not contribute to the amplitude since they cannot be contracted with α^-_n. This makes the calculation very simple and indeed we find that the amplitude is the same as that of usual D-brane in the flat space. To see this, note that there is no zero-mode contribution to A^i_m $(m \neq 0)$ in (2.16). Almost the same result can also be obtained for traveling waves of transverse scalar fields $\phi_i^{(1)}(x^+)$ and $\phi_i^{(2)}(x^+)$. The only difference is that we have an additional factor $\exp[-\frac{2\pi^2\alpha'}{s}(\phi_i^{(1)}(x^+) - \phi_i^{(2)}(x^+))^2]$, which represents the time dependent winding energy between the two different D-branes (for details, see (3.21) in section 3.3).

These simple results (and their analogous results of closed strings in plane wave backgrounds[6]) correspond to the stringy version of the known fact that there are no particle creations and vacuum polarization in Yang-Mills or gravitational plane wave background [21, 20]. However, things will be different due to winding modes if we compactify a spatial coordinate in the light-cone direction on a circle as we will see in the next section.

3. D-brane with compactified traveling waves

As we have seen above, the vacuum amplitude between D-branes with traveling waves in flat space turns out to be trivial. On the other hand, if we study the open string spectrum in the light-cone gauge [14], then we get the non-trivial time dependent world-sheet dynamics. This is because the boundary interaction $\int d\tau A^i(X^+)\partial_\tau X_i$ becomes time dependent linear interaction after we impose the light-cone gauge $X^+ = x^+ + 2\alpha' p^+ \tau$ with non-zero p^+. Why is there such a difference between these two analyses of the same system? The answer is that the state with non-zero p^+ cannot be (easily) expressed in the boundary state formalism since in the closed string channel the non-zero p^+ sector corresponds to the non-zero winding sector, which does not exist in our non-compact space analysis.

[6]In the exactly solvable plane wave with NSNS-flux (Nappi-Witten model [8]), it has been known that the partition function is the same as that in the flat space [30, 31]. For the plane wave background with RR-field [3], a similar result seems to be difficult to show since there is no solvable covariant formalism (see [32] for a relevant discussion in the operator formalism in the light-cone gauge). Nevertheless, there are some evidences for the triviality of partition function constant [33, 34]. On the other hand, after we compactify a spatial coordinate $y(=\frac{1}{2}x^+ - x^-)$, the partition function can be non-trivial as known in the NSNS plane wave [31] (see also [35] for DLCQ compactification of plane wave). Indeed our results of open string analogue of plane waves have a similar property as we will show later.

In order to see the time dependent effects from the closed string viewpoint, we compactify the y direction (here we assume the coordinates $x^+ = t+y$ and $x^- = (t-y)/2$) and study the corresponding boundary state. Mainly we consider the traveling waves of gauge fields on the spacetime filling brane (D25-brane) in bosonic string theory. Later we will also give the results in the transverse scalar case and superstring case briefly. We assume that the gauge field $A_i(X^+)$ obeys the periodicity

$$A^i(X^+ + 2\pi R) = A^i(X^+)\,, \tag{3.1}$$

which allows the Fourier expansion ($c_n = c^*_{-n}$)

$$A^i(X^+) = \sum_{n=-\infty}^{\infty} c^i_n e^{-i\frac{nX^+}{R}}\,. \tag{3.2}$$

3.1 Vacuum amplitude

Let us consider the vacuum amplitude between two such spacetime filling D-branes. Since the operators α^+_n ($n \neq 0$) commute with each other, we again conclude that only zero-modes on A^i_m in the boundary state (2.16) contribute to the vacuum amplitude. By dividing (3.2) into zero-modes x^+, w and massive modes $\alpha^+_{\pm n}$, we obtain

$$A^i(X^+) = \sum_{n=-\infty}^{\infty} \left(c^i_n e^{-i\frac{nx^+}{R}} e^{-2inw\sigma} \right) + \text{massive terms}\,. \tag{3.3}$$

In the non-compact case (i.e., $w = 0$), it is easy to see that the zero-mode term in A^i_m is zero except for $m = 0$ as we have seen in the previous section. Thus we could effectively regard the boundary state as the usual one $|N\rangle$ in the computation of vacuum amplitude.

However, in the compactified case an interesting thing does happen. The zero-mode terms in A^i_m are given by

$$A^i_{nw} = c^i_n e^{-i\frac{nx^+}{R}}\,. \tag{3.4}$$

Thus in the computation of vacuum amplitude we effectively obtain a novel form of boundary state, neglecting the massive modes α^+_n ($n \neq 0$),

$$\begin{aligned}|P\rangle \sim \exp\Bigg(& -4\pi^2\alpha'|w| \sum_{n=1}^{\infty} n|c_n|^2 + \\ & + 2\sqrt{2\alpha'}\pi i \sum_{n\geq 1} \left(c^i_n \alpha^i_{-n|w|} e^{-i\frac{nx^+}{R}\frac{w}{|w|}} - c^i_{-n}\tilde{\alpha}^i_{-n|w|} e^{i\frac{nx^+}{R}\frac{w}{|w|}} \right) \Bigg)|N\rangle\,, \end{aligned} \tag{3.5}$$

which gives a different weight to each winding sector. (Notice that the sum over w is implicitly hidden in $|N\rangle$.) The presence of the exponential factor $\sim \exp(-|w|)$ leads to the suppression for the sectors with large winding number. This will be interpreted as the damping behavior of open string in the time dependent background. The second term in the exponential in (3.5) represents the linear interaction of open string, which is obvious from the form of the boundary interaction.

Now let us compute the amplitude between a D25-brane with gauge flux $A^{(1)}(x^+)$ and another with $A^{(2)}(x^+)$. Employing the formula

$$\begin{aligned}&\langle 0|e^{-\frac{1}{m}\alpha_m\tilde{\alpha}_m}e^{(f^{(2)}_{-m}\alpha_m+f^{(2)}_{m}\tilde{\alpha}_m)}e^{-sH_c}e^{(f^{(1)}_{m}\alpha_{-m}+f^{(1)}_{-m}\tilde{\alpha}_{-m})}e^{-\frac{1}{m}\alpha_{-m}\tilde{\alpha}_{-m}}|0\rangle = \\ &= \frac{1}{1-e^{-2ms}}\exp\left(-\frac{m}{e^{2ms}-1}\Big[|f^{(1)}_m|^2+|f^{(2)}_m|^2-e^{ms}(f^{(1)}_m f^{(2)}_{-m}+f^{(1)}_{-m}f^{(2)}_m)\Big]\right), \quad (3.6)\end{aligned}$$

which can be shown by repeatedly applying the Baker-Campbell-Hausdorff's formula ($f^{(1,2)}_m$ $(=f^{(1,2)*}_{-m})$ are arbitrary constants), we obtain[7]

$$\begin{aligned}Z &= \langle P^{(2)}|\Delta|P^{(1)}\rangle \\ &= \mathcal{N}\int_0^\infty ds\sum_{w\in\mathbb{Z}}\exp\Big[-\frac{w^2R^2s}{2\alpha'}\Big]\frac{e^{2s}}{\prod_{m=1}^\infty(1-e^{-2ms})^{24}}\times \\ &\quad\times\exp\Bigg[-\sum_{i=1}^{24}\sum_{n=1}^{\infty}\frac{4\pi^2\alpha' wn}{e^{2nws}-1}\Big((|c^{(1)i}_n|^2+|c^{(2)i}_n|^2)(e^{2nws}+1)- \\ &\qquad\qquad -2(c^{(1)i}_n c^{(2)i}_{-n}+c^{(1)i}_{-n}c^{(2)i}_n)e^{nws}\Big)\Bigg]. \qquad (3.7)\end{aligned}$$

The oscillator contributions from X^+ and X^- are canceled by that from the bc ghosts. The normalization factor $\mathcal{N}$ is given by $\mathcal{N}=\frac{\alpha' T_{25}^2}{8}V_{26}$ with the volume of spacetime $V_{26}=\int dx^+dx^-dx^1\cdots dx^{24}$. Note also that the last exponential in (3.7) is always less than one since it can be written as

$$\exp\left[-\sum\frac{4\pi^2\alpha' wn}{e^{2nws}-1}\Big(|e^{nws}c^{(1)i}_n-c^{(2)i}_n|^2+|e^{nws}c^{(2)i}_n-c^{(1)i}_n|^2\Big)\right]. \qquad (3.8)$$

In particular, if we consider the amplitude between the same D-brane $c^{(1)i}_n=c^{(2)i}_n(\equiv c^i_n)$, then the non-trivial factor is simplified as

$$\exp\left[-8\pi^2\alpha' w\sum_{i=1}^{24}\sum_{n=1}^{\infty}n|c^i_n|^2\Big(\frac{e^{nws}-1}{e^{nws}+1}\Big)\right]. \qquad (3.9)$$

It is also possible to generalize these results to the vacuum amplitude between two such D-branes in superstring theory. We can employ a similar argument on decoupling of massive modes, and in conclusion we only have to replace the modular function $\frac{e^{2s}}{\prod_{m=1}^\infty(1-e^{-2ms})^{24}}\equiv\eta(is/\pi)^{-24}$ in (3.7) by the familiar terms with theta-functions

$$\frac{\theta_3(is/\pi)^4-\theta_2(is/\pi)^4-\theta_4(is/\pi)^4}{2\eta(is/\pi)^{12}}. \qquad (3.10)$$

Therefore the vacuum amplitude vanish due to the supersymmetry. This is consistent with the fact that each of the D-branes with traveling waves preserve the same eight supersymmetries. If one wants to see a similar non-trivial time dependent effect even in superstring theory, he or she should consider a brane-antibrane system. The vacuum amplitude can be obtained by just changing the sign in front of $\theta_2(is/\pi)^4$ in (3.10). In this case the amplitude does not vanish and a non-trivial interaction is left.

[7]Here we have replaced $|w|$ in (3.5) with w using a symmetry in the final expression of amplitude.

3.2 Physical interpretation of vacuum amplitude

Now let us consider the physical interpretations of our vacuum amplitude (3.7), which represents the interaction between two D-branes with traveling waves. In many familiar examples, we can perform modular transformation $s = \pi/t$ and express the amplitude in the open string channel (see also (2.6)). In our case, however, the modular transformation seems very difficult to perform. Actually, this is natural because we know that the world-sheet theory is time dependent (for non-zero p^+) in the open string side, and the open string cylinder amplitude should become complicated. Nevertheless, we can extract some information from our amplitude computed in the boundary state formalism by taking the IR limit $s \to 0$ of the open string (on the other hand, $s \to \infty$ corresponds to the IR limit of closed string).

First let us consider the IR limit $s \to \infty$ of closed string (UV limit $t \to 0$ in open string side). In this case the important exponential factor becomes

$$\sim \exp\left[-4\pi^2\alpha'|w|\sum_{i=1}^{24}\sum_{n=1}^{\infty} n(|c_n^{(1)i}|^2 + |c_n^{(2)i}|^2)\right]. \tag{3.11}$$

Thus we have no interaction between the two waves of gauge fields $A^{(1)}$ and $A^{(2)}$ since there is no mixing term like $c_n^{(1)i}c_{-n}^{(2)i}$. This is natural because the closed string propagates for a long distance. Note also that the interaction for the winding sectors is suppressed by the presence of the waves on D-branes. In the case with a strong pulse $\sum_{n=1}^{\infty} n|c_n|^2 \gg (\alpha')^{-1}$, no winding mode will propagate between the D-branes.

On the other hand, in the UV limit $s \to 0$ of closed string (IR limit $t \to \infty$ of open string), the interaction of two waves becomes strong as can be seen from the non-trivial factor (for $w \neq 0$)

$$\sim \exp\left[-4\pi\alpha' t\sum_{i=1}^{24}\sum_{n=1}^{\infty} |c_n^{(1)i} - c_n^{(2)i}|^2\right]. \tag{3.12}$$

This means that the interaction is suppressed for the winding sectors except for the case $c_n^{(1)i} = c_n^{(2)i}$ (the interaction between the same D-brane). This is much like a mass shift $\delta m^2 \sim 2\sum_{n=1}^{\infty}|c_n^{(1)i} - c_n^{(2)i}|^2$ due to the time dependent Wilson-lines.

Consider the interaction between the same D-brane. Then, the contribution from the leading order (3.12) vanishes and the first non-trivial correction to (3.12) is given at the order $\sim O(w^2 s)$. Combining the first factor in (3.7), this correction can be regarded as the shift of radius

$$R'^2 = R^2 + \frac{4}{3}\pi^2\alpha'^2\sum_{n=1}^{\infty} n^2\left(2|c_n^{(1)}|^2 + 2|c_n^{(2)}|^2 + c_n^{(1)}c_{-n}^{(2)} + c_{-n}^{(1)}c_n^{(2)}\right). \tag{3.13}$$

Indeed the shifted radius can be interpreted as the one defined by the open string metric $G_{\mu\nu} = g_{\mu\nu} - (2\pi\alpha')^2 F_{\mu\alpha}g^{\alpha\beta}F_{\beta\nu}$ (see, for example, [36]). The corrected radius is estimated as

$$R'^2 - R^2 = (2\pi\alpha' R)^2\langle(F_{+i})^2\rangle = 8(\pi\alpha')^2\sum_{n=1}^{\infty} n^2|c_n|^2\,, \tag{3.14}$$

which agrees with (3.13). Here we take the average $\langle\cdots\rangle$ over x^+ in the last equality. After performing the modular transformation, the mass spectrum of open string includes the canonical Kaluza-Klein momentum term n^2/R'^2 for the shifted radius.

It would also be interesting to ask what will happen if we take $R \to 0$ limit. For the usual D25-brane (i.e., $c_n = 0$), we will obtain a D24-brane by T-duality, under which the winding mode is identified with the momentum such that $P_{\tilde{y}} = wR$. The momentum $P_{\tilde{y}}$ could be finite under the limit $R \to 0$ by taking $w \to \infty$ with keeping the combination wR finite. On the other hand, if we naively T-dualize our D25-brane with traveling waves in the limit, then we get a 'exotic' D24-brane with only $P_{\tilde{y}} = 0$ sector in the boundary state since the contributions from the large w sectors are suppressed as mentioned around eq. (3.5). This means that the D-brane configuration is smeared along the y direction (this might be natural since the pulse originally runs in the y direction). However, in a physical theory, only finite energy configurations of gauge fields are allowed. This condition is estimated (assuming a weak gauge field) as follows[8] (for a finite gauge coupling)

$$E \propto \int dy \Big[(F_{0i})^2 + (F_{ij})^2\Big] \propto \frac{1}{R}\sum_n n^2|c_n|^2 \,. \tag{3.15}$$

Therefore we can see that a D25-brane with a finite energy pulse becomes an ordinary D24-brane after T-duality transformation.

3.3 D-strings with traveling waves

As we mentioned above, the configurations considered are T-dual to D-strings with waves traveling at the speed of light, thus the boundary states for these D-strings are obtained by T-dualizing the boundary states we have constructed. In order to express the Dirichlet boundary state, it is convenient to use the following coherent state

$$|p\rangle = \prod_{m\geq 1} \exp\left[-\frac{1}{m}\alpha_{-m}\tilde{\alpha}_{-m} + \frac{p_m}{m}\alpha_{-m} + \frac{p_{-m}}{m}\tilde{\alpha}_{-m} - \frac{p_m p_{-m}}{2m}\right]|0\rangle \,, \tag{3.16}$$

which satisfies

$$(\alpha_m + \tilde{\alpha}_{-m} - p_m)|p\rangle = 0 \,, \tag{3.17}$$

where we have imposed $p_m = p^*_{-m}$. In this basis the Neumann boundary state is given by $|p=0\rangle \otimes |p=0\rangle_{(0)}$ with ${}_{(0)}\langle p|p'\rangle_{(0)} = \delta(p-p')$ and the Dirichlet boundary state $|D1, x^i\rangle$ can be written as

$$\begin{aligned} |D1, x^i\rangle &= \prod_{i=1}^{24}\int dp^i \prod_{m\geq 1} dp^i_m dp^i_{-m}|p^i\rangle \otimes e^{-ip^i x^i}|p^i\rangle_{(0)} \otimes |N_{lc}\rangle \\ &= \prod_i \prod_{m\geq 1} \exp\left(\frac{1}{m}\alpha^i_{-m}\tilde{\alpha}^i_{-m}\right)|0\rangle \otimes \int dp^i e^{-ip^i x^i}|p^i\rangle_{(0)} \otimes |N_{lc}\rangle \,, \end{aligned} \tag{3.18}$$

where $|N_{lc}\rangle$ represents the Neumann boundary state for light-cone directions. This is nothing but the expression of Dirichlet boundary state in the momentum basis.

[8]We may also get stronger conditions from next order ($\mathcal{O}(F^4)$) terms.

By using the basis, we can write the T-dual version of (2.12) as

$$
\begin{aligned}
|P\rangle &= \mathrm{P}\prod_i \exp\left(-i\int_0^\pi d\sigma \phi^i(X^+)\partial_\tau X^i\right)|D1,0\rangle \\
&= \prod_i \exp\left(\sum_{m=1}^{\infty}(-4\pi^2\alpha' m\phi^i_m\phi^i_{-m} - 2\sqrt{2\alpha'}\pi i\phi^i_m\alpha^i_{-m} - 2\sqrt{2\alpha'}\pi i\phi^i_{-m}\tilde{\alpha}^i_{-m})\right)\times \\
&\quad \times \exp\left(\sum_n \frac{1}{n}\alpha^i_{-n}\tilde{\alpha}^i_{-n}\right)|0\rangle\otimes\int dp^i e^{-2\pi i\alpha' p^i\phi^i_0(X^+))}|p^i\rangle_{(0)}\otimes|N_{lc}\rangle\,, \qquad (3.19)
\end{aligned}
$$

where

$$
\phi^i(X^+) = \sum_n \phi^i_n(X^+)e^{-2in\sigma}\,. \qquad (3.20)
$$

We should remark that the position of D-string is shifted by the zero mode $\phi^i_0(X^+)$, which also includes constant shift. The amplitude between D-strings with pulses can be calculated as

$$
\begin{aligned}
Z &= \langle P^{(2)}|\Delta|P^{(1)}\rangle \\
&= \mathcal{N}\int_0^\infty ds\int dx^+\left(\frac{2\pi}{\alpha' s}\right)^{12} e^{-\frac{2\pi^2\alpha'}{s}\sum_i\left(\phi^{(1)i}_0(x^+)-\phi^{(2)i}_0(x^+)\right)^2}\times \\
&\quad\times\sum_{w\in\mathbb{Z}}\exp\Big[-\frac{w^2R^2s}{2\alpha'}\Big]\frac{e^{2s}}{\prod_{m=1}^\infty(1-e^{-2ms})^{24}}\times \\
&\quad\times\exp\Bigg[-\sum_{i=1}^{24}\sum_{n=1}^{\infty}\frac{4\pi^2\alpha'|w|n}{e^{2n|w|s}-1}\Big((|c^{(1)i}_n|^2+|c^{(2)i}_n|^2)(e^{2n|w|s}+1)- \\
&\qquad\qquad -2(c^{(1)i}_n c^{(2)i}_{-n}+c^{(1)i}_{-n}c^{(2)i}_n)e^{n|w|s}\Big)\Bigg]\,. \qquad (3.21)
\end{aligned}
$$

The non-zero modes of X^+ do not contribute to ϕ^i_0 in the final expression as before, and hence the amplitude depends on the distance between two D-branes in the usual way.

4. D-brane with more general gauge fields

In the presence of non-trivial gauge fields, the open string metric can be written as

$$
G_{\mu\nu} = \eta_{\mu\nu} - (2\pi\alpha')^2 F_{\mu\rho}\eta^{\rho\sigma}F_{\sigma\nu}\,, \qquad (4.1)
$$

as mentioned above. We have already dealt with the case of $F_{+i} = h_i(x^+)$, and we try to extend our result to the case of more general gauge field in this section. When $F_{+i} = h_{ij}(x^+)x^j$ ($\sum_i h_{ii} = 0$) is included, the corresponding open string metric becomes

$$
ds^2 = -2dx^+dx^- - \sum_{i,j,k} h_{ij}(x^+)h_{ik}(x^+)x^jx^k(dx^+)^2 + \sum_i (dx^i)^2\,, \qquad (4.2)
$$

which is the metric of the time dependent plane wave type.[9]

[9] As argued in [15], we can include the field strength of the form $F_{+i}(x^+,x^i)$ with preserving 1/4 supersymmetry, and the configuration preserves also the conformal symmetry if the gauge field satisfies $\partial_j\partial^j A_+(x^+x^i) = 0$. For example, the two cases $F_{+i} = h_i(x^+)$ and $F_{+i} = h_{ij}(x^+)x^j$ ($\sum_i h_{ii} = 0$) satisfy the condition.

We only consider bosonic string theory and D25-brane with field strength $F_{+i} = 2h_i(x^+)x^i$ ($\sum_i h_i(x^+) = 0$) for simplicity. Then, the boundary state for the D-brane can be written by acting Wilson line to the Neumann boundary states as

$$\begin{aligned} |P\rangle &= \mathrm{P}\exp\left(-i\int_0^\pi d\sigma A_+(X^+,X^i)\partial_\sigma X^+\right)|N\rangle \\ &= \mathrm{P}\exp\left(-i\int_0^\pi d\sigma \sum_i h_i(X^+)\partial_\sigma X^+ X^i X^i\right)|N\rangle \,. \end{aligned} \tag{4.3}$$

Then the vacuum amplitude[10] in non-compact space becomes trivial as before. Thus let us again assume y direction is compactified $X^+ + 2\pi R \sim X^+$.

Since there is a periodicity under $\sigma \to \sigma + \pi$, we can expand as

$$h_i(X^+)\partial_\sigma X^+ = \sum_n H_n^i(X^+)e^{-2in\sigma}\,. \tag{4.4}$$

By inserting this mode expansion into the previous Wilson line, we can proceed the calculation as

$$\begin{aligned} |P\rangle = &\int \prod_i dx^i \prod_{m\geq 1} dx_m^i dx_{-m}^i \times \\ &\times \exp\left(-i\pi H_0^i x^i x^i + \sqrt{2\alpha'}\pi\sum_{n\neq 0} H_{-n}^i \frac{x_n^i}{n} x^i - i\pi\frac{\alpha'}{2}\sum_{m,n\neq 0}\frac{x_m^i}{m}H_{-m+n}^i\frac{x_{-n}^i}{n}\right)\times \\ &\times \exp\left(\frac{\tilde{\alpha}_{-m}^i\alpha_{-m}^i}{m} + \frac{x_m^i\alpha_{-m}}{m} - \frac{x_{-m}^i\tilde{\alpha}_{-m}^i}{m} - \frac{x_m^i x_{-m}^i}{m}\right)|0\rangle\otimes|x^i\rangle_{(0)}\otimes|N_{lc}\rangle\,, \end{aligned} \tag{4.5}$$

where we use the equality up to normalization. In this expression, we can perform the gaussian integral for x_m^i with $m \neq 0$ and obtain

$$\begin{aligned} |P\rangle = &\prod_i (\det\Delta_{mn}^i)^{-1}\prod_{m,p,n\neq 0}\exp\left(-\frac{1}{2m}a_m^i\left(\epsilon(m)\delta_{m,p} - \frac{2\pi i\alpha'}{p}H_{-m+p}^i\right)(\Delta^i)_{pn}^{-1}a_{-n}^i\right)|0\rangle\otimes \\ &\otimes\int dx^i\prod_{p,q\neq 0}\exp\Bigg(-i\pi H_0^i x^i x^i - 2\alpha'\pi^2\frac{H_p^i}{p}(\Delta^i)_{pq}^{-1}H_{-q}^i x^i x^i - \\ &\qquad - \sqrt{2\alpha'}\pi x^i\left(\frac{H_p^i}{p}(\Delta^i)_{pq}^{-1}a_{-q}^i + \frac{a_p^i}{p}(\Delta^i)_{pq}^{-1}H_{-q}^i\right)\Bigg)|x^i\rangle_{(0)}\otimes|N_{lc}\rangle\,. \end{aligned} \tag{4.6}$$

We have used $a_{-m}^i = \alpha_m^i$, $a_m^i = \tilde{\alpha}_{-m}^i$ for $m \geq 1$ and defined

$$\Delta_{mn}^i = \epsilon(m)\delta_{m,n} + \frac{2\pi i\alpha'}{n}H_{-m+n}^i\,, \tag{4.7}$$

where $\epsilon(m)$ represents the sign of m. We should notice that this expression is similar to the boundary state with a constant flux [37]. Although it is straightforward to calculate the

[10] Some results of cylinder amplitude in the light-cone gauge can be found in [15].

amplitudes between the boundary states or closed string states, it seems that the results cannot be summarized in a simple form. Thus, we study the amplitudes in a simpler case in the rest of this section.

When $h_i(X^+) = \mu_i$ ($\sum_i \mu_i = 0$), equivalently

$$H_0^i = 2Rw\mu_i \equiv \frac{w\nu_i}{2\pi\alpha'}\,, \qquad H_n^i = 0 \quad (n \neq 0)\,, \tag{4.8}$$

the open string metric becomes that of a time *in*dependent plane wave. In this case, the boundary state (4.6) can be written in a simple form as

$$\begin{aligned}|\vec{\nu}\rangle = &\prod_i \prod_{m\geq 1} \left(1 + \frac{i\nu_i w}{m}\right)^{-1} \exp\left(-\frac{1}{m}\alpha^i_{-m}\left(\frac{m - i\nu_i w}{m + i\nu_i w}\right)\tilde{\alpha}^i_{-m}\right)|0\rangle \otimes \\ &\otimes \int dx^i \exp\left(-\frac{iw\nu_i x^i x^i}{2\alpha'}\right)|x^i\rangle_{(0)} \otimes |N_{lc}\rangle\,. \end{aligned} \tag{4.9}$$

The amplitude between this type of boundary states is given by

$$\begin{aligned} Z &= \langle \vec{\nu}^{(2)}|\Delta|\vec{\nu}^{(1)}\rangle \\ &= \int_0^\infty ds e^{2s} \sum_w \exp\left[-\frac{w^2R^2 s}{2\alpha'}\right] \prod_i \left(\frac{2\pi s w^2 \nu_i^{(1)}\nu_i^{(2)}}{\alpha'} + \frac{2\pi i w\left(\nu_i^{(1)} - \nu_i^{(2)}\right)}{\alpha'}\right)^{-1/2} \times \\ &\times \prod_{m\geq 1}\left(\left(1 + \frac{i\nu_i^{(1)}w}{m}\right)\left(1 - \frac{i\nu_i^{(2)}w}{m}\right) - \left(1 - \frac{i\nu_i^{(1)}w}{m}\right)\left(1 + \frac{i\nu_i^{(2)}w}{m}\right)e^{-2sm}\right)^{-1}. \end{aligned} \tag{4.10}$$

It would be interesting if we can apply the modular transformation to this amplitude and interpret it in a open string channel.

5. Interaction of pulse and anti-pulse

Next let us proceed to a more complicated and intriguing example, i.e., the interaction between two waves of gauge field $A^i(X^+)$ and $\tilde{A}^i(X^-)$ traveling in the opposite directions (see figure 1). In contrast with the previous examples, we expect non-trivial particle creation in this example since there is no symmetry in the null direction. Thus this configuration leads to a more interesting time dependent effect.

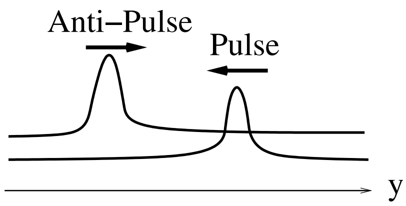

Figure 1: The collision of pulse and anti-pulse. In this figure we consider the T-dualized case, i.e., the pulse-like waves of transverse scalars on two D-strings.

Since we can use any form of gauge fields $A^i(X^+)$ and $\tilde{A}^i(X^-)$, we choose the ones of pulse-like form. In this case we can regard this physical setup as a collision between pulse P^+ and anti-pulse P^-. In superstring theory, P^+ and P^- preserve different types of eight supersymmetries, and hence the presence of both leads to a non-supersymmetric system. Thus this system is expected to be unstable and will tend to decay when the two pulses are approaching. If the decay occurs completely, it will eventually becomes the supersymmetric system of two overlapped D-branes with no gauge flux. However, we cannot deny the possibility that the annihilation of the pulse and anti-pulse takes place partially and smaller pulses remain.

The vacuum amplitude of this system can be computed in our boundary state formalism and is non-trivial even in non-compact space as we will see below. Another motivation to study this system is that the system is the open string analogue of collision of plane-waves as it is difficult to compute in closed string. Also it seems impossible to compute it in open string in the light-cone gauge. Thus this example is an interesting application of our boundary state formulation.

5.1 D-branes with constant null flux

Before we discuss the general system, we would like to examine a 'toy model', i.e., two (spacetime filling) D-branes $|F^+\rangle$ and $|F^-\rangle$ with constant gauge flux F_{+i} and F_{-i}, respectively (or $A_i(X^\pm) = F_{\pm i}X^\pm$). Essentially cylinder amplitude has already been computed in [38] by using the analytic continuation of the light-cone boundary state (see also [39] for open string spectrum and refer to [40] for an earlier literature) in the context of BPS brane-antibrane system [41]. Here we will examine it by using the covariant boundary state and clarify the physical phenomena especially for non-supersymmetric cases.

Note that we cannot reduce this system to well-known case of (purely) magnetic or electric flux [37, 22] by Lorentz transformation. For simplicity we assume that only $f_+ \equiv 2\pi\alpha' F_{+1}$ and $f_- \equiv 2\pi\alpha' F_{-1}$ are non-zero. Then, the boundary states $|F^\pm\rangle$ should satisfy the boundary conditions

$$\begin{aligned}(\partial_\tau X^\mp - f_\pm \partial_\sigma X^i)|F^\pm\rangle &= 0\,,\\ \partial_\tau X^\pm |F^\pm\rangle &= 0\,,\\ (\partial_\tau X^i - f_\pm \partial_\sigma X^\pm)|F^\pm\rangle &= 0\,.\end{aligned} \tag{5.1}$$

These conditions are solved as in (2.12) (we only write $|F^+\rangle$)

$$\begin{aligned}|F^+\rangle &= \exp\left[\sum_{n=1}^{\infty}\frac{2}{n}\Big(f_+^2\alpha^+_{-n}\tilde\alpha^+_{-n} - f_+\alpha^1_{-n}\tilde\alpha^+_{-n} + f_+\alpha^+_{-n}\tilde\alpha^1_{-n}\Big)\right]|N\rangle \qquad (5.2)\\ &= \frac{T_{25}}{2}\prod_{n=1}^{\infty}\frac{n}{2\pi}\int d\lambda_n d\lambda_n^*\exp\Big[-\frac{n}{2}|\lambda_n|^2 - \lambda_n(f_+\tilde\alpha^+_{-n} + \tilde\alpha^1_{-n}) - \lambda_n^*(f_+\alpha^+_{-n} - \alpha^1_{-n}) +\\ &\qquad + \frac{1}{n}\Big(\alpha^+_{-n}\tilde\alpha^-_{-n} + \alpha^-_{-n}\tilde\alpha^+_{-n} + \alpha^i_{-n}\tilde\alpha^i_{-n}\Big)\Big]|0\rangle \otimes \int d\vec{x}\;|\vec{x}\rangle_{(0)}\,.\end{aligned}$$

Using the second integral expression in (5.2), we can compute the vacuum amplitude Z_f between $|F^+\rangle$ and $|F^-\rangle$ as follows;

$$\begin{aligned}Z_f &= \mathcal{N}\int_0^\infty ds\; e^{2s}\prod_{n=1}^{\infty}\frac{1}{(1-e^{-2ns})^{24}}\cdot\Big(\frac{n}{2\pi}\Big)^2\int d\lambda_n d\lambda_n^* d\mu_n d\mu_n^* \times\\ &\times \exp\left[-\frac{n(e^{2ns}+1)}{2(e^{2ns}-1)}(|\lambda_n|^2 + |\mu_n|^2) + \frac{ne^{ns}(1-f_+f_-)}{e^{2ns}-1}(\lambda_n\mu_n^* + \mu_n\lambda_n^*)\right]\end{aligned}$$

$$= \mathcal{N}\int_0^\infty ds\ e^{2s}\prod_{n=1}^{\infty}\frac{1}{(1-e^{-2ns})^{24}}\cdot\frac{(1-e^{-2ns})^2}{(1-2\cos(2\pi\nu)e^{-2ns}+e^{-4ns})}\,, \tag{5.3}$$

where we define

$$\cos(2\pi\nu) = 2(f_+f_- - 1)^2 - 1\,. \tag{5.4}$$

Furthermore, it is possible to write the amplitude in terms of eta- and theta-functions as

$$Z_f = 2\mathcal{N}\sin(\pi\nu)\int_0^\infty ds\ \frac{1}{\eta(\frac{is}{\pi})^{21}\ \theta_1(\nu|\frac{is}{\pi})}\,. \tag{5.5}$$

After performing the modular transformation ($t = \pi/s$) we obtain

$$\begin{aligned} Z_f &= -2i\mathcal{N}\sin(\pi\nu)\int_0^\infty dt\ \frac{\pi e^{\pi\nu^2 t}}{t^{13}}\cdot\frac{1}{\eta(it)^{21}\ \theta_1(-i\nu t|it)} \qquad (5.6)\\ &= i\mathcal{N}\sin(\pi\nu)\int_0^\infty dt\ \frac{\pi e^{\pi(2+\nu^2)t}}{t^{13}\sin(\pi i\nu t)}\prod_{n=1}^{\infty}\frac{1}{(1-e^{-2\pi nt})^{22}(1-e^{-2\pi nt+2\pi\nu t})(1-e^{-2\pi nt-2\pi\nu t})}\,. \end{aligned}$$

The supersymmetrization is also easy to be done (recall the Jacobi identity)

$$\begin{aligned} Z_f &= \frac{\mathcal{N}\sin(\pi\nu)}{8\sin(\pi\nu/2)^4}\int_0^\infty ds\ \frac{\theta_1(\frac{\nu+\epsilon}{2}|\frac{is}{\pi})^4}{\eta(\frac{is}{\pi})^8\ \theta_1(\nu|\frac{is}{\pi})}\\ &= \frac{-i\mathcal{N}\pi\sin(\pi\nu)}{\sin(\pi\nu/2)^4}\int_0^\infty dt\ \frac{e^{-\pi t(\epsilon^2+2\nu\epsilon)}}{8t^5}\cdot\frac{\theta_1(-i\frac{(\nu+\epsilon)t}{2}|it)^4}{\eta(it)^8\ \theta_1(-i\nu t|it)}\,, \qquad (5.7) \end{aligned}$$

where we should set $\epsilon = 0$ for brane-brane amplitude and $\epsilon = 1$ for brane-antibrane one. We can also check that the above results (5.6) and (5.7) in terms of modular parameter t agree with the open string spectrum (see [39] for open string computations for $f_+ = f_-$ case).

We can see from (5.6) and (5.7) that there are infinite number of poles from the factor $\sin(2\pi i\nu t)$ for imaginary values of ν ($(f_+f_- - 1)^2 > 1$). This is very similar to the physics on D-branes with (purely) electric-field [22]. Thus, for imaginary ν, the integration over t leads to the imaginary part of the amplitude, and hence open string pair creations should occur. Notice that ν takes an imaginary value when $f_+f_- < 0$. This is intuitively natural because the electric fields on two D-branes have the opposite sign. On the other hand, ν takes a real value for $0 < f_+f_- < 2$ and the spectrum includes open string tachyons induced by the gauge flux. Thus open string tachyon condensation should occur in this case. In the other case $f_+f_- > 2$, ν becomes imaginary and we observe pair creations.

At the one critical point $f_+f_- = 0$ ($\nu = 0$) the spectrum is the same as the usual D-brane since the gauge field F_{+i} (or F_{-i}) does not polarize the vacuum as we have seen above. We also have the simplified spectrum at the other critical point $f_+f_- = 2$ ($\nu = 1$). The amplitude (5.7) between branes ($\epsilon = 0$) is the same as that of a brane-antibrane without gauge flux, while that between brane and antibrane ($\epsilon = 1$) is the same as the usual supersymmetric amplitude between branes. The latter corresponds to (a generalization of) the fact found in [41] that a brane-antibrane system with the critical electric flux and opposite sign of magnetic flux becomes supersymmetric.

In summary, the system of two D-branes with constant flux f_+ and f_- is unstable in general and should decay via either open string pair productions or open string tachyon condensation.[11]

5.2 Pulse and anti-pulse scattering

Now let us turn to the main issue of computing the interaction (or equally vacuum amplitude) of pulse and anti-pulse. For simplicity we show only the result in bosonic strings. The result is not changed substantially even in superstring theory. The boundary state for a pulse $|P^+\rangle$ is given by (2.12) and that for an anti-pulse $|P^-\rangle$ is simply given by replacing α_n^+ in (2.12) with α_n^-. For anti-pulse, we denote the integration as $dy_m^\mu dy_{-m}^\mu$ to avoid a confusion. Then, the vacuum amplitude is defined as

$$Z_{+-} = \langle P_-|\Delta|P_+\rangle\,, \tag{5.8}$$

which becomes a rather non-trivial amplitude since we should take infinitely many contractions of α_n^+ and α_{-n}^- in the Wilson-line terms. After performing the integration over x_m^i and y_m^i in (5.8), we obtain

$$\begin{aligned}
Z_{+-} = \mathcal{N}' \int_0^\infty ds\, & \frac{e^{2s}}{\prod_{m=1}^\infty (1-e^{-2ms})^{24}} \int dx^+ dy^- \prod_{m\geq 1} dx_m^\pm dx_{-m}^\pm dy_m^\pm dy_{-m}^\pm \times \\
& \times \exp\Bigg[-\frac{4\pi^2\alpha' m(e^{2ms}+1)}{(e^{2ms}-1)} \Big(A_{-m}^i(\hat{x}^+) A_m^i(\hat{x}^+) + \tilde{A}_{-m}^i(\hat{y}^-)\tilde{A}_m^i(\hat{y}^-)\Big) + \\
& \qquad + \frac{8\pi^2\alpha' m e^{ms}}{(e^{2ms}-1)} \Big(A_{-m}^i(\hat{x}^+)\tilde{A}_m^i(\hat{y}^-) + \tilde{A}_{-m}^i(\hat{y}^-) A_m^i(\hat{x}^+)\Big) \Bigg] \times \\
& \times \exp\Bigg[\frac{1}{m(e^{2ms}-1)} \Big(\frac{e^{2ms}+1}{2} (x_m^+ x_{-m}^- + x_m^- x_{-m}^+ + y_m^+ y_{-m}^- + y_m^- y_{-m}^+) - \\
& \qquad - e^{ms}(x_m^+ y_{-m}^- + x_{-m}^+ y_m^- + x_m^- y_{-m}^+ + x_{-m}^- y_m^+)\Big)\Bigg]\,,
\end{aligned} \tag{5.9}$$

where the normalization $\mathcal{N}'$ is defined such that $\mathcal{N} = \mathcal{N}' \int dx^+ dy^-$.

Finally we integrate out x_m^- and y_m^+ as follows;

$$\begin{aligned}
Z_{+-} = \mathcal{N}' \int_0^\infty ds\, & \frac{e^{2s}}{\prod_{m=1}^\infty (1-e^{-2ms})^{24}} \times \\
& \times \int dx^+ dy^- \prod_{m\geq 1} dx_m^+ dx_{-m}^+ dy_m^- dy_{-m}^- \times (e^{ms}-e^{-ms})^2 \times \\
& \times \exp\left[\frac{e^{2ms}-1}{4me^{ms}} (x_m^+ y_{-m}^- + x_{-m}^+ y_m^-)\right] \times \\
& \times \exp\Bigg[-\frac{4\pi^2\alpha' m(e^{2ms}+1)}{(e^{2ms}-1)} \Big(A_{-m}^i(\hat{x}^+) A_m^i(\hat{x}^+) + \tilde{A}_{-m}^i(\hat{y}^-)\tilde{A}_m^i(\hat{y}^-)\Big) + \\
& \qquad + \frac{8\pi^2\alpha' m e^{ms}}{(e^{2ms}-1)} \Big(A_{-m}^i(\hat{x}^+)\tilde{A}_m^i(\hat{y}^-) + \tilde{A}_{-m}^i(\hat{y}^-) A_m^i(\hat{x}^+)\Big) \Bigg]\,.
\end{aligned} \tag{5.10}$$

[11] Recently it was argued in [42] that the open string tachyon condensation may also lead to another kind of open string pair productions.

Unfortunately it is difficult to perform the integrations in (5.10). Therefore let us take the slowly changing gauge field limit. Then we obtain

$$Z_{+-} \sim \mathcal{N}' \int_0^\infty ds \, \frac{e^{2s}}{\prod_{m=1}^\infty (1-e^{-2ms})^{24}} \int dx^+ dy^- \, \partial_+ A_i(x^+) \partial_- \tilde{A}_i(y^-) \sum_{n=1}^\infty \frac{16\pi^2 \alpha'^2 e^{2ns}}{(e^{2ns}-1)^2} \,. \tag{5.11}$$

In the large s limit we can regard this as the closed string exchange between two D-branes. For example, the contribution from $m = 1$ or $n = 1$ part in (5.11) represents a massless field exchange.

Furthermore, we can also get the full order result with respect to $\alpha'^2 \partial_+ A_i(x^+) \partial_- \tilde{A}_i(y^-)$ by neglecting higher derivatives, and the result is simply given by the previous formula (5.5). We should note that the value of ν depends on x^+, y^- via $F_{+i}(x^+)$, $F_{-i}(y^-)$ and we should also make the integration $\int dx^+ dy^-$ explicit. Thus, in this approximation, the previous result of toy model (with only constant flux) can be utilized. Since the pulses, in general, have both positive and negative values of gauge flux depending on the time and position, the collision of pulse and anti-pulse may lead to both open string creation and open string tachyon condensation. These phenomena happen when the pulses approach, and the both effects should cause the decay of the system at least partially. A part of the energy may be carried out by the radiations (closed strings). It is an interesting future problem to find the exact end point of this unstable system for arbitrary pulses.

6. Traveling tachyonic waves

In a brane-antibrane system, there is an open string tachyon field and we can consider D-brane configuration with a non-trivial tachyon profile [23]. It was shown in [43] that a non-BPS D-brane can be described by a tachyonic kink on a brane-antibrane pair by using conformal field theory and later it was confirmed in [44] by using boundary state formalism.[12] In the previous analysis, we only include the non-trivial gauge fields depending on one of the light-cone directions x^+. The configuration with x^+ dependent tachyon is also an interesting system, which we analyze in this section. As the S-brane [48] or rolling tachyon [49, 29] gives an important time dependent system in string theory, our traveling tachyonic wave (or 'null tachyon') may lead to another one.

Let us consider D9-brane and anti-D9-brane wrapped on a torus with radii R for y direction and R^1 for x^1 direction and include $\mathbb{Z}_2$ Wilson line on the anti-D9-brane. In this configuration the tachyon on a open string stretched between two branes has an anti-periodic boundary condition and an mode expansion

$$T(x^+, x^1) = \sum_n T_{n+1/2}(x^+) e^{i \frac{n+1/2}{R^1} x^1} \,. \tag{6.1}$$

We have assumed that the tachyon has no dependence of x^- (and also the transverse directions except for x^1 direction). When we restrict the radius to $R^1 = \sqrt{\alpha'/2}$, the

[12] A tachyon vortex [45] on the brane-antibrane pair as a marginal deformation was discussed in [46] by using boundary state description. See also, e.g., [47] for off-shell boundary state description of open string tachyon condensation.

tachyon of the form

$$T(x^+,x^1) = \frac{1}{\sqrt{2}} t(x^+) \cos\left(\sqrt{\frac{2}{\alpha'}}(x^1 - Y^1(x^+))\right) \tag{6.2}$$

becomes an exactly marginal operator for any $t(x^+)$ and $Y^1(x^+)$.

It is known that when $t(x^+) = 1/2$ and $Y^1(x^+) = 0$, the D9-brane anti-D9-brane pair with the tachyonic kink is equivalent to a non-BPS D8-brane [43]. The position of the non-BPS D8-brane corresponds to the point of $T(x^+,x^1) = 0$, thus, in particular, the configuration with $t(x^1) = 1/2$ and non-zero $Y^1(x^+)$ describes non-BPS D8-brane at $x^1 = Y^1(x^+)$ just like the D-strings in subsection 3.3. From now on we set $Y^1(x^+) = 0$ for simplicity.

The D9-branes on a torus with radii R and R^1 in the type IIB superstring theory can be described by the boundary states

$$|N\rangle_{\rm NS} = \frac{1}{2}[|N,+\rangle_{\rm NS} - |N,-\rangle_{\rm NS}]\,, \qquad |N\rangle_{\rm R} = \frac{1}{2}[|N,+\rangle_{\rm R} + |N,-\rangle_{\rm R}]\,, \tag{6.3}$$

and the ghost part. The explicit form in the NSNS sector is given by

$$\begin{aligned} |N,\pm\rangle_{\rm NS} = \int d\vec{x} \sum_{w,w^1} \exp\left(-\sum_{m\geq 1} \frac{1}{m} \alpha^\mu_{-m} g_{\mu\nu} \tilde{\alpha}^\nu_{-m}\right) \times \\ \times \exp\left(\pm i \sum_{r\geq 1/2} \psi^\mu_{-r} g_{\mu\nu} \tilde{\psi}^\nu_{-r}\right) |\vec{x}, w, w^1\rangle_{\rm NS}\,, \end{aligned} \tag{6.4}$$

where w^1 is the winding number for x^1 direction. (We concentrate on the NSNS-sector in this section.) Then, the pair of D9-brane and anti-D9-brane with $\mathbb{Z}_2$ Wilson line can be described by the following boundary states

$$|B,\pm\rangle_{\rm NS} = |N,\pm\rangle_{\rm NS} + |N',\pm\rangle_{\rm NS}\,, \qquad |B,\pm\rangle_{\rm R} = |N,\pm\rangle_{\rm R} - |N',\pm\rangle_{\rm R}\,, \tag{6.5}$$

where $|N',\pm\rangle$ includes the $\mathbb{Z}_2$ Wilson line.

When $R^1 = \sqrt{\alpha'/2}$, we can fermionize the boson $X^1(\tau,\sigma) = \frac{1}{2}(X^1(\tau+\sigma) + \tilde{X}^1(\tau-\sigma))$ as

$$\begin{aligned} e^{\pm\frac{i}{\sqrt{2\alpha'}}X^1(\tau+\sigma)} &\simeq \frac{1}{\sqrt{2}}(\eta(\tau+\sigma) \pm i\xi(\tau+\sigma))\,, \\ e^{\pm\frac{i}{\sqrt{2\alpha'}}\tilde{X}^1(\tau-\sigma)} &\simeq \frac{1}{\sqrt{2}}(\tilde{\eta}(\tau-\sigma) \pm i\tilde{\xi}(\tau-\sigma))\,. \end{aligned} \tag{6.6}$$

By using the fermionic partners ψ^1 and $\tilde{\psi}^1$, we can define a new bosonic field $\phi(\tau,\sigma) = \frac{1}{2}(\phi(\tau+\sigma) + \tilde{\phi}(\tau-\sigma))$ by

$$\begin{aligned} \frac{1}{\sqrt{2}}(\xi(\tau+\sigma) \pm i\psi^1(\tau+\sigma)) &\simeq e^{\pm\frac{i}{\sqrt{2\alpha'}}\phi(\tau+\sigma)}\,, \\ \frac{1}{\sqrt{2}}(\tilde{\xi}(\tau-\sigma) \pm i\tilde{\psi}^1(\tau-\sigma)) &\simeq e^{\pm\frac{i}{\sqrt{2\alpha'}}\tilde{\phi}(\tau-\sigma)}\,. \end{aligned} \tag{6.7}$$

The advantage of the redefinition of coordinates is that we can rewrite the tachyon vertex in a simple form (in the zero picture) as [43]

$$V_T = \frac{it(X^+)}{\sqrt{2\alpha'}} \partial_\sigma \phi(\sigma) \otimes \sigma^1 \,. \tag{6.8}$$

The sigma matrix σ^1 corresponds to the Chan-Paton factor.

In the new coordinate system with ϕ and η $(\tilde{\eta})$, the Neumann boundary state can be constructed by replacing X^1 and ψ^1 by ϕ and η [44, 46] as

$$\begin{aligned} |B,\pm\rangle_{\rm NS} = \sum_{w,w^\phi} \int \prod_{\rho,\mu,\nu\neq 1} dx^\rho dx^\phi \exp\left(-\sum_{m\geq 1} \frac{1}{m} \alpha^\mu_{-m} g_{\mu\nu} \tilde{\alpha}^\nu_{-m}\right) \exp\left(\pm i \sum_{r\geq 1/2} \psi^\mu_{-r} g_{\mu\nu} \tilde{\psi}^\nu_{-r}\right) \times \\ \times \exp\left(-\sum_{m\geq 1} \frac{1}{m} \alpha^\phi_{-m} \tilde{\alpha}^\phi_{-m}\right) \exp\left(\pm i \sum_{r\geq 1/2} \eta_{-r} \tilde{\eta}_{-r}\right) |x^\rho, x^\phi, w, 2w^\phi\rangle_{\rm NS} \,. \end{aligned} \tag{6.9}$$

The mode expansions of ϕ and η are given in the way similar to X^1 and ψ^1. Using the tachyon vertex operator (6.8), we obtain the boundary state for D9-brane anti-D9-brane pair with the tachyonic kink as

$$\begin{aligned} |T,\pm\rangle_{\rm NS} &= \frac{1}{2} \mathrm{PTr} \exp\left(i \int_0^\pi d\sigma \frac{t(X^+)}{\sqrt{2\alpha'}} \partial_\sigma \phi(\sigma) \otimes \sigma^1\right) |B,\pm\rangle_{\rm NS} \\ &= \frac{1}{2} \exp\left(-2\pi^2 \sum_{m=1}^\infty m t_m t_{-m}\right) \Big[\exp\left(2\pi i (t_m \alpha^\phi_{-m} - t_{-m} \tilde{\alpha}^\phi_{-m} + t_0 w^\phi)\right) + \\ &\qquad + \exp\left(-2\pi i (t_m \alpha^\phi_{-m} - t_{-m} \tilde{\alpha}^\phi_{-m} + t_0 w^\phi)\right)\Big] |B,\pm\rangle_{\rm NS} \,, \end{aligned} \tag{6.10}$$

where we defined the mode expansion as $t(X^+) = \sum_n t_n e^{-2in\sigma}$. Therefore, we conclude that the configuration with a traveling open string tachyon can be analyzed in the way similar to the ones with traveling gauge fields. For example, the amplitudes between these boundary states can be calculated as we have done in (3.7).

7. Summary and conclusions

In this paper we have investigated several properties of D-branes with traveling waves in the covariant boundary state formalism. The traveling waves are carried by gauge fields or transverse scaler fields. Interestingly, the boundary states have a novel feature that they have infinitely many parameters, which describe every forms of traveling waves.

Employing the boundary states and computing their vacuum amplitudes, we analyzed the interactions between these D-branes. We found that in non-compact spacetime the interactions are the same as those between usual D-branes. However, in the compactified case the interactions turn out to be very non-trivial and they depend on the form of the waves explicitly. The non-trivial contribution comes from winding modes of closed strings in the compact space, as we can see it directly in the boundary state. In this way we found that the time dependence affects the interaction between D-branes with traveling waves if we compactify the space in the traveling direction.

We also generalized the form of waves such that the waves depend on the spatial coordinate other than x^+, and obtained the vacuum amplitude in a simplified case. In the example, the open string metric on the D-brane becomes that of a plane-wave not of a general pp-wave.

By using our formalism it is also possible to compute the interaction between two D-branes with waves traveling in the opposite direction; the configuration seems difficult to analyze in the light-cone gauge. This leads to an interesting non-supersymmetric time dependent system in superstring theory. In particular, we can choose the form of wave such that it represents the collision of pulse and anti-pulse. We obtained the integral formula of the vacuum amplitude and argued that the open string creation or tachyon condensation may occur and it may lead to a (partial) decay of this unstable system.

Finally we considered the application of these traveling wave configurations to the open string tachyon condensation. We obtained a new boundary state which represents the traveling tachyonic waves.

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A. Pulse D-brane as boundary deformation

In this appendix we would like to discuss D-branes with traveling waves from the point of view of the boundary deformation [50]. When we consider the (normal) Neumann boundary condition, the gluing condition is given by

$$\partial X^\mu(z) = \bar{\partial} X^\mu(\bar{z}) \qquad \text{at} \qquad z = \bar{z}\,, \tag{A.1}$$

where the world-sheet is the upper half plane of z and the boundary is $\mathrm{Im}(z) = 0$. The OPE between bulk fields is given by

$$X^\mu(z_1,\bar{z}_1) X^\nu(z_2,\bar{z}_2) = -\frac{\alpha'}{2}\eta^{\mu\nu}\left[\ln|z_1 - z_2|^2 + \ln|z_1 - \bar{z}_2|^2\right] + \text{reg}\,, \tag{A.2}$$

and we also obtain the OPE between boundary fields by setting $z_i = \bar{z}_i = x_i$,

$$X^\mu(x_1) X^\nu(x_2) = -2\alpha' \eta^{\mu\nu} \ln|x_1 - x_2| + \text{reg}\,. \tag{A.3}$$

Next let us deform the gluing condition (A.1) to that with traveling waves

$$\partial_\tau X^i - 2\pi\alpha' \partial_\sigma A^i(X^+) = 0 \qquad \text{at} \qquad \tau = 0\,, \tag{A.4}$$

where relations between (σ,τ) and $(z,\bar{z})$ in this appendix are given by

$$z = (\sigma + \tau), \quad \bar{z} = (\sigma - \tau)\,, \qquad \partial = \frac{1}{2}(\partial_\sigma + \partial_\tau)\,, \qquad \bar{\partial} = \frac{1}{2}(\partial_\sigma - \partial_\tau)\,. \tag{A.5}$$

As we will find, the deformation is generated by the boundary marginal field $\mathcal{A}$

$$\mathcal{A}(\sigma) \equiv -iA_i(X^+(\sigma))\partial X^i(\sigma) = i\int \mathrm{d}k^- c_i(k^-)e^{-ik^- X^+(\sigma)}\partial X^i(\sigma)\,, \tag{A.6}$$

where the Fourier mode $c_i(k^-)$ satisfies $c_i^*(k^-) = c(-k^-)$. Notice that the operator $\mathcal{A}$ is self-local (in the terminology of [50]) because the OPE between $\mathcal{A}$'s is given by

$$\mathcal{A}(x_1)\mathcal{A}(x_2) = \frac{2\alpha' A_i(x_1)A^i(x_2)}{(x_1-x_2)^2} + \mathrm{reg}\,. \tag{A.7}$$

A.1 Boundary conformal invariance

In order to preserve boundary conformal symmetry under the deformation, it is necessary that there is no mixing between the deformation field $\mathcal{A}$ and any marginal field. Thus it is needed that the following three point functions for all marginal fields ψ vanish

$$\langle \mathcal{A}(x_1)\mathcal{A}(x_2)\psi(x_3)\rangle = 0\,. \tag{A.8}$$

By using the momentum conservation and the contraction of ∂X^i, we can trivially show that eq. (A.8) is satisfied for all marginal fields except $\psi = \partial X^-$. Thus we only need to calculate the boundary three point function

$$\langle \mathcal{A}(x_1)\mathcal{A}(x_2)\partial X^-(x_3)\rangle \equiv \frac{C_{\mathcal{AA}P^-}}{(x_1-x_2)(x_2-x_3)(x_3-x_1)}\,. \tag{A.9}$$

Using the correlation function

$$\begin{aligned}
&\langle : e^{-ik_1^- X^+(x_1)} :: e^{-ik_2^- X^+(x_2)} : \partial X^i(x_1)\partial X^j(x_2)\partial X^-(x_3)\rangle_N = \\
&\quad = iC\delta(k_1^- + k_2^-)\Big[2i\alpha'\Big(\frac{k_1^-}{x_3-x_1} + \frac{k_2^-}{x_3-x_2}\Big)\Big]\frac{(-2\alpha'\delta^{ij})}{(x_1-x_2)^2} \\
&\quad = -\frac{4\alpha'^2 C\delta^{ij}\delta(k_1^-+k_2^-)k_1^-}{(x_1-x_2)(x_2-x_3)(x_3-x_1)}\,,
\end{aligned} \tag{A.10}$$

with a constant C, we obtain

$$\begin{aligned}
C_{\mathcal{AA}P^-} &= 4\alpha'^2 \int \mathrm{d}k_1^-\,\mathrm{d}k_2^-\, c_i(k_1^-)c_i(k_2^-)\delta(k_1^-+k_2^-)k_1^- \\
&= \frac{2\alpha'^2}{\pi}\int \mathrm{d}y \int \mathrm{d}k_1^-\,\mathrm{d}k_2^-\, c_i(k_1^-)c_i(k_2^-)k_1^- e^{-iy(k_1^-+k_2^-)} \\
&= \frac{2i\alpha'^2}{\pi}\int \mathrm{d}y \frac{d}{dy}A_i^2(y) = 0\,.
\end{aligned} \tag{A.11}$$

Thus we conclude that eq. (A.8) is satisfied for all marginal fields.

A.2 Boundary deformation

Finally we show that the marginal field $\mathcal{A}$ deforms the (normal) Neumann condition to that with traveling waves (A.4). The gluing condition is deformed by the deformation as follows [50]

$$0 = \lim_{\delta\to+0}\sum_{n=0}^{\infty}\frac{1}{n!}\int_{\gamma_1}\cdots\int_{\gamma_n}\mathrm{d}\sigma_1\cdots\mathrm{d}\sigma_n \mathcal{A}(\sigma_1)\cdots\mathcal{A}(\sigma_n)[\partial X^i(z_\delta) - \bar\partial X^i(\bar z_\delta)]\,, \tag{A.12}$$

where we define $z_\delta = x + 2i\delta$, $\bar{z}_\delta = x - 2i\delta$ and γ_p as the line $\mathrm{Im}(\sigma_p) = \frac{\epsilon}{p}$ $(\epsilon < \delta)$. Notice that we have regularized eq. (A.12) by analytically continuing $\mathcal{A}(\sigma)$ into the upper half plane. Since the self local operator $\mathcal{A}$ has the important property

$$\int_{\gamma_1} \mathrm{d}z \int_{\gamma_2} \mathrm{d}z' \mathcal{A}(z)\mathcal{A}(z') = 0\,, \tag{A.13}$$

where we assume that there is no insertion in $\mathrm{Re}(z) > 0$, there is no contribution from the $n > 1$ part of (A.12). Thus we have to examine only the $n = 1$ part, which is calculated as

$$\int_{-\infty}^{\infty} \mathrm{d}\sigma \mathcal{A}(\sigma)\partial X^j(z_2) = -2\pi\alpha'\partial A^i(\tilde{X}^+(z_2))\,, \qquad \int_{-\infty}^{\infty} \mathrm{d}\sigma \mathcal{A}(\sigma)\bar{\partial} X^j(\bar{z}_2) = 0\,, \tag{A.14}$$

where we defined $\tilde{X}^+(z) \equiv \frac{1}{2}(X_L^+(z) + X_R^+(\bar{z}))$. Therefore the deformed boundary condition is obtained as follows

$$\begin{aligned} 0 &= \lim_{\delta\to+0} \left[1 + \int_{-\infty}^{\infty} \mathrm{d}\sigma \mathcal{A}(\sigma)\right] [\partial X^i(z_\delta) - \bar{\partial} X^i(\bar{z}_\delta)] \\ &= \partial_\tau X^i(x) - 2\pi\alpha'\partial_+ A^i(X^+(x))\partial_\sigma X^+(x)\,. \end{aligned} \tag{A.15}$$

This is nothing but the condition (A.4).

References

[1] D. Amati and C. Klimčík, *Strings in a shock wave background and generation of curved geometry from flat space string theory*, *Phys. Lett.* **B 210** (1988) 92;
G.T. Horowitz and A.R. Steif, *Space-time singularities in string theory*, *Phys. Rev. Lett.* **64** (1990) 260;
G.T. Horowitz and A.R. Steif, *Strings in strong gravitational fields*, *Phys. Rev.* **D 42** (1990) 1950;
H.J. de Vega and N. Sanchez, *Space-time singularities in string theory and string propagation through gravitational shock waves*, *Phys. Rev. Lett.* **65** (1990) 1517.

[2] M. Blau, J. Figueroa-O'Farrill, C. Hull and G. Papadopoulos, *A new maximally supersymmetric background of IIB superstring theory*, *J. High Energy Phys.* **01** (2002) 047 [hep-th/0110242].

[3] R.R. Metsaev, *Type IIB Green-Schwarz superstring in plane wave Ramond-Ramond background*, *Nucl. Phys.* **B 625** (2002) 70 [hep-th/0112044];
R.R. Metsaev and A.A. Tseytlin, *Exactly solvable model of superstring in plane wave Ramond-Ramond background*, *Phys. Rev.* **D 65** (2002) 126004 [hep-th/0202109].

[4] D. Berenstein, J.M. Maldacena and H. Nastase, *Strings in flat space and PP waves from $N = 4$ super Yang-Mills*, *J. High Energy Phys.* **04** (2002) 013 [hep-th/0202021].

[5] G. Papadopoulos, J.G. Russo and A.A. Tseytlin, *Solvable model of strings in a time-dependent plane-wave background*, *Class. and Quant. Grav.* **20** (2003) 969 [hep-th/0211289];
M. Blau and M. O'Loughlin, *Homogeneous plane waves*, *Nucl. Phys.* **B 654** (2003) 135 [hep-th/0212135].

JHEP04(2003)032

[6] G.T. Horowitz and A.R. Steif, *Singular string solutions with nonsingular initial data*, *Phys. Lett.* **B 258** (1991) 91.

[7] J. Simon, *The geometry of null rotation identifications*, *J. High Energy Phys.* **06** (2002) 001 [hep-th/0203201];
H. Liu, G. Moore and N. Seiberg, *Strings in a time-dependent orbifold*, *J. High Energy Phys.* **06** (2002) 045 [hep-th/0204168]; *Strings in time-dependent orbifolds*, *J. High Energy Phys.* **10** (2002) 031 [hep-th/0206182];
A. Lawrence, *On the instability of 3d null singularities*, *J. High Energy Phys.* **11** (2002) 019 [hep-th/0205288];
M. Fabinger and J. McGreevy, *On smooth time-dependent orbifolds and null singularities*, hep-th/0206196;
G.T. Horowitz and J. Polchinski, *Instability of spacelike and null orbifold singularities*, *Phys. Rev.* **D 66** (2002) 103512 [hep-th/0206228].

[8] C.R. Nappi and E. Witten, *A WZW model based on a nonsemisimple group*, *Phys. Rev. Lett.* **71** (1993) 3751 [hep-th/9310112].

[9] J.G. Russo and A.A. Tseytlin, *On solvable models of type-IIB superstring in NS-NS and R-R plane wave backgrounds*, *J. High Energy Phys.* **04** (2002) 021 [hep-th/0202179];
E. Kiritsis and B. Pioline, *Strings in homogeneous gravitational waves and null holography*, *J. High Energy Phys.* **08** (2002) 048 [hep-th/0204004];
A. Parnachev and D.A. Sahakyan, *Penrose limit and string quantization in $AdS_3 \times S^3$*, *J. High Energy Phys.* **06** (2002) 035 [hep-th/0205015];
Y. Hikida and Y. Sugawara, *Superstrings on PP-wave backgrounds and symmetric orbifolds*, *J. High Energy Phys.* **06** (2002) 037 [hep-th/0205200];
Y. Hikida and Y. Sugawara, *Superstring vacua of 4-dimensional PP-waves with enhanced supersymmetry*, *J. High Energy Phys.* **10** (2002) 067 [hep-th/0207124];
H. Kunitomo, *Hybrid superstrings in NS-NS plane waves*, hep-th/0303039.

[10] N. Berkovits, *Conformal field theory for the superstring in a Ramond-Ramond plane wave background*, *J. High Energy Phys.* **04** (2002) 037 [hep-th/0203248].

[11] C.G. Callan and I.R. Klebanov, *D-brane boundary state dynamics*, *Nucl. Phys.* **B 465** (1996) 473 [hep-th/9511173].

[12] C. Bachas and C. Hull, *Null brane intersections*, *J. High Energy Phys.* **12** (2002) 035 [hep-th/0210269].

[13] K. Skenderis and M. Taylor, *Open strings in the plane wave background, I. Quantization and symmetries*, hep-th/0211011.

[14] C. Bachas, *Relativistic string in a pulse*, hep-th/0212217.

[15] B. Durin and B. Pioline, *Open strings in relativistic ion traps*, hep-th/0302159.

[16] C.G. Callan Jr., J.M. Maldacena and A.W. Peet, *Extremal black holes as fundamental strings*, *Nucl. Phys.* **B 475** (1996) 645 [hep-th/9510134];
A. Dabholkar, J.P. Gauntlett, J.A. Harvey and D. Waldram, *Strings as solitons & black holes as strings*, *Nucl. Phys.* **B 474** (1996) 85 [hep-th/9511053].

[17] D. Mateos, S. Ng and P.K. Townsend, *Supercurves*, *Phys. Lett.* **B 538** (2002) 366 [hep-th/0204062].

[18] R.C. Myers and D.J. Winters, *From D-$\bar{D}$ pairs to branes in motion*, *J. High Energy Phys.* **12** (2002) 061 [hep-th/0211042].

JHEP04(2003)032

[19] K. Okuyama, *D-branes on the null-brane*, *J. High Energy Phys.* **02** (2003) 043 [hep-th/0211218].

[20] G.W. Gibbons, *Quantized fields propagating in plane wave space-times*, *Commun. Math. Phys.* **45** (1975) 191.

[21] S. Deser, *Plane waves do not polarize the vacuum*, *J. Phys.* **A 8** (1975) 1972.

[22] C. Bachas and M. Porrati, *Pair creation of open strings in an electric field*, *Phys. Lett.* **B 296** (1992) 77 [hep-th/9209032].

[23] A. Sen, *Tachyon condensation on the brane antibrane system*, *J. High Energy Phys.* **08** (1998) 012 [hep-th/9805170].

[24] C.G. Callan Jr., C. Lovelace, C.R. Nappi and S.A. Yost, *Loop corrections to superstring equations of motion*, *Nucl. Phys.* **B 308** (1988) 221.

[25] J.L. Cardy, *Boundary conditions, fusion rules and the Verlinde formula*, *Nucl. Phys.* **B 324** (1989) 581.

[26] P. Di Vecchia and A. Liccardo, *D-branes in string theory, I*, hep-th/9912161; *D-branes in string theory, II*, hep-th/9912275.

[27] P. Di Vecchia, M. Frau, I. Pesando, S. Sciuto, A. Lerda and R. Russo, *Classical p-branes from boundary state*, *Nucl. Phys.* **B 507** (1997) 259 [hep-th/9707068].

[28] P. Di Vecchia, M. Frau, A. Lerda and A. Liccardo, *(F, Dp) bound states from the boundary state*, *Nucl. Phys.* **B 565** (2000) 397 [hep-th/9906214].

[29] A. Sen, *Tachyon matter*, *J. High Energy Phys.* **07** (2002) 065 [hep-th/0203265].

[30] E. Kiritsis, C. Kounnas and D. Lüst, *Superstring gravitational wave backgrounds with space-time supersymmetry*, *Phys. Lett.* **B 331** (1994) 321 [hep-th/9404114].

[31] J.G. Russo and A.A. Tseytlin, *Constant magnetic field in closed string theory: an exactly solvable model*, *Nucl. Phys.* **B 448** (1995) 293 [hep-th/9411099].

[32] T. Takayanagi, *Modular invariance of strings on PP-waves with RR-flux*, *J. High Energy Phys.* **12** (2002) 022 [hep-th/0206010].

[33] A. A. Tseytlin, unpublished note.

[34] A.B. Hammou, *One loop partition function in plane waves RR background*, *J. High Energy Phys.* **11** (2002) 028 [hep-th/0209265].

[35] Y. Sugawara, *Thermal amplitudes in DLCQ superstrings on PP-waves*, *Nucl. Phys.* **B 650** (2003) 75 [hep-th/0209145].

[36] N. Seiberg and E. Witten, *String theory and noncommutative geometry*, *J. High Energy Phys.* **09** (1999) 032 [hep-th/9908142].

[37] A. Abouelsaood, C.G. Callan Jr., C.R. Nappi and S.A. Yost, *Open strings in background gauge fields*, *Nucl. Phys.* **B 280** (1987) 599.

[38] D. Mateos, S. Ng and P.K. Townsend, *Tachyons, supertubes and brane/anti-brane systems*, *J. High Energy Phys.* **03** (2002) 016 [hep-th/0112054].

[39] D.-S. Bak and N. Ohta, *Supersymmetric D2-anti-D2 strings*, *Phys. Lett.* **B 527** (2002) 131 [hep-th/0112034].

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[40] B.M. Barbashov, A.L. Koshkarov and V.V. Nesterenko, *Relativistic string in the constant homogeneous electromagnetic field*, *Teor. Mat. Fiz.* **32** (1977) 176.

[41] D.S. Bak and A. Karch, *Supersymmetric brane-antibrane configurations*, *Nucl. Phys.* **B 626** (2002) 165 [hep-th/0110039].

[42] A. Strominger, *Open string creation by S-branes*, hep-th/0209090.

[43] A. Sen, SO(32) *spinors of type-I and other solitons on brane-antibrane pair*, *J. High Energy Phys.* **09** (1998) 023 [hep-th/9808141].

[44] M. Frau, L. Gallot, A. Lerda and P. Strigazzi, *Stable non-BPS D-branes in type-I string theory*, *Nucl. Phys.* **B 564** (2000) 60 [hep-th/9903123].

[45] J. Majumder and A. Sen, *Vortex pair creation on brane-antibrane pair via marginal deformation*, *J. High Energy Phys.* **06** (2000) 010 [hep-th/0003124].

[46] M. Naka, T. Takayanagi and T. Uesugi, *Boundary state description of tachyon condensation*, *J. High Energy Phys.* **06** (2000) 007 [hep-th/0005114].

[47] T. Asakawa, S. Sugimoto and S. Terashima, *Exact description of D-branes via tachyon condensation*, *J. High Energy Phys.* **02** (2003) 011 [hep-th/0212188].

[48] M. Gutperle and A. Strominger, *Spacelike branes*, *J. High Energy Phys.* **04** (2002) 018 [hep-th/0202210].

[49] A. Sen, *Rolling tachyon*, *J. High Energy Phys.* **04** (2002) 048 [hep-th/0203211].

[50] A. Recknagel and V. Schomerus, *Boundary deformation theory and moduli spaces of D-branes*, *Nucl. Phys.* **B 545** (1999) 233 [hep-th/9811237].

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